Chapter 1
Application of the Impedance Matched Multi-Axis Testing Theme of Smart Dynamic Testing

Michael T. Hale, William A. Barber, and Jacob A. Davis

Abstract Execution of vibration testing without imparting major impedance mismatches between the field and laboratory test environments has long been a challenge in the dynamic test community. The historic technique of designing vibration fixtures such that the first mode is beyond the maximum frequency of the test is based on the false premise that the vibration levels imparted to the unit under test (UUT) are uniform at all input points. Testing in a serial single degree-of-freedom (SDOF), further diverges the classical laboratory test from the true nature of the multiple degree-of-freedom (MDOF) tactical field environment. The use of a stiff cradle fixture to address captive carry or free-flight environments of a missile system is a prime example of a test configuration that severely restricts the UUT's natural modal characteristics, and the resulting mechanical impedance has no correlation to the field conditions. The Impedance Matched Multi-Axis Testing (IMMAT) theme of Smart Dynamic Testing (SDT), aims to provide a reasonable representation of the global vibration response of operating environments such as those that are aerodynamic or acoustic in nature (e.g., captive carriage, free-flight vibration). This paper will discuss IMMAT and associated SDT themes and illustrate the concepts through a laboratory-based application on a dynamically accurate missile surrogate using a reconfigurable test stand.

Keywords Dynamic · Impedance · Multi-shaker · Modal · Test

Background

The traditional approach of the use of rigid fixturing attached to a high impedance SDOF shaker for replicating aerodynamic environments in the laboratory is known to be poor. This has been recognized by dynamicist and test engineers for many years; however, in the early years of dynamic testing there were test equipment limitations that restricted testing options [1]. In recent years, significant advances have been made in instrumentation, test hardware and control systems that have opened the possibilities for more advanced test approaches. Daborn, Ind and Ewins proposed implementation of multiple small exciters to conduct MDOF testing on a freely suspended structure [2]. Roberts and Ewins focused further on the concepts of impedance matching and MDOF testing, demonstrating significant improvements over traditional SDOF approaches [3]. The early work with the use of multiple small shakers and impedance optimization by Daborn, Ind, Roberts and Ewings led to the term Impedance Matched Multi-Axis Testing (IMMAT). Daborn used the phrase "Smarter Dynamic Testing" in his dissertation [4].

Professor David Ewins was a champion of the work referenced above, and he worked tirelessly with the dynamic test community in an effort to formalize the approach [5]. A Workshop and informal Community of Practice (CoP) was

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established in 2018 and the term "Smart Dynamic Testing" (SDT) was adopted. Understanding the risks of unrealistic laboratory conditions and opportunities to merge test and analysis was recognized by all participants as keys to future test planning and development [6].

During the 2018 Workshop, the underlying technologies that form the basis of SDT were discussed, and these were presented as 4 themes which provide a well-defined structure for future development of new testing techniques and of their eventual documentation and reference. The Technology Themes identified were:

I. Test Specification: Environmental capture and definition.
II. Boundary Conditions and Fixture Design.
III. Excitation Strategy (IMMAT).
IV. Test Optimization and Integration

While engineering associated with each of the four themes is common, developing a more synergetic test approach integrating across all themes is the objective of SDT.

The Redstone Test Center's (RTC) Dynamic Test Division has the primary mission of providing vibration, acoustics, mechanical shock and pyrotechnic shock testing for the US Army and DoD at large. The lab has a wide range of traditional electrodynamic and servo hydraulic excitation systems in both SDOF and MDOF configurations. In recent years RTC and other Dynamic Test facilities have begun to integrate SDT concepts into the test paradigm. The presented example is intended to illustrate a typical SDT approach as related to captive carriage of a typical fixed wing aircraft store.

The example UUT, as illustrated in Figure 1, is characteristic of a typical aircraft store for which captive carry vibration testing will be included as part of the qualification program. Due to the sensitivity of the test payload, detailed pictures and specific engineering units were not included in this document.



Fig. 1 Five Shaker IMMAT Setup

SDT theme I (Test specification)

For the example UUT, the objectives were to conduct captive carry buffet and non-buffet vibration tests. Ideally, the product of Theme I, will be a fully populated Spectral Density Matrix (SDM) based on field measurements that would serve as the test reference criteria. Auto Spectral Density (ASD) reference criteria in each of the three mutually orthogonal directions were provided at six locations along the length of the test item. Unfortunately, and is often the case, Cross-Spectral Density (CSD) profiles were not provided. The alternate approach is to utilize the modal characteristics in the test configuration to compute a CSD. In these scenarios, reproducing all 18 ASDs within traditional MIL-STD-810 tolerances is highly unlikely and success depends greatly on the enveloping or averaging methods utilized in the original specification development.

SDT theme II (Boundary conditions and fixture design)

Mechanical Setup

It is acknowledged that the best impedance match to aircraft captive carry boundary conditions would consist of the UUT attached to an isolated launcher assembly with the isolation frequency tuned to the first bending mode of the aircraft wing. However, limitations of launcher availability often restrict this approach and require alternative levels of attachment to the vibration source. As discussed in Reference [1], each alternative comes with a level of compromise with respect to impedance, but some are certainly better than others. For example, the historically acceptable practice of conducting a vibration test by placing the UUT in a cradle fixture significantly influences the natural modal characteristics of the test article yielding structural responses with minimal correlation to that of the fielded configuration. Unfortunately, such practices continue today across the industry. Other techniques such as exciting the UUT with a single shaker stinger assembly at one end of the payload have limitations in that only the portion of the UUT nearest the drive point would come close to the desired reference criteria. The use of dual exciters for a single axis test with some level of rotational relief (e.g., stingers, radial bearings) at the attachment points has shown significant improvement for captive carry vibration especially when coupled with a launcher interface, but this approach is typically limited to a single translational axis of vibration and does not fully capture the effects of the coupled degrees of freedom. All the techniques described above employ large, fixed base high force exciters with large spatial footprints that are heavy and not practical to configure for multiple translational DOF excitation.

For the test example, a launcher was not available and the UUT was simply supported with elastic cordage. This setup is most representative of free-free boundary conditions, although analysis has shown that the captive carry boundary conditions are not significantly different from the free-free boundary conditions. It is acknowledged that this mechanical setup does not induce loads through the launcher interface, but this was not an objective of this particular test.

Test Fixtures

The collar fixtures that allowed connection of the stingers to the UUT were repurposed from previous dual shaker trials. Due to the relatively large mass of the collars (approximately 10 pounds each), the free-free modes of the UUT were reduced by approximately 5% as shown in Figures 2 & 3 and Table 3. While not ideal, they provide a rigid connection to the UUT across the bandwidth of interest and were considered sufficient for initial trials.

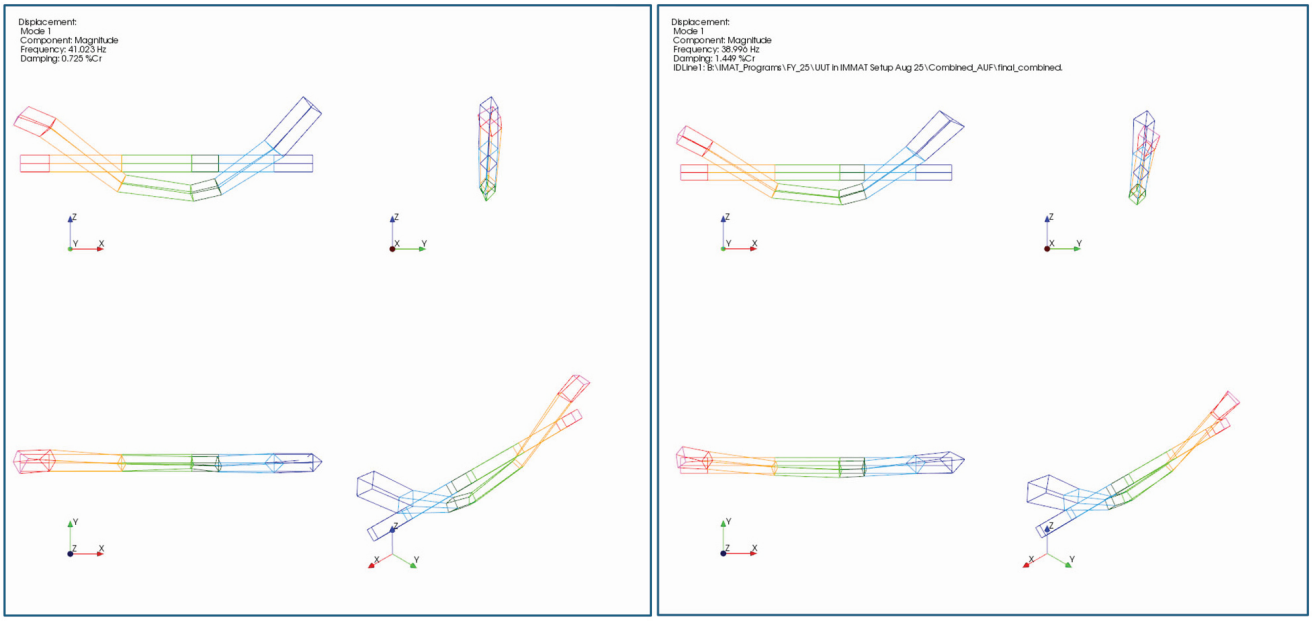


Fig. 2 Left: Free-Free: Mode 1. Right: Configured for IMMAT: Mode 1

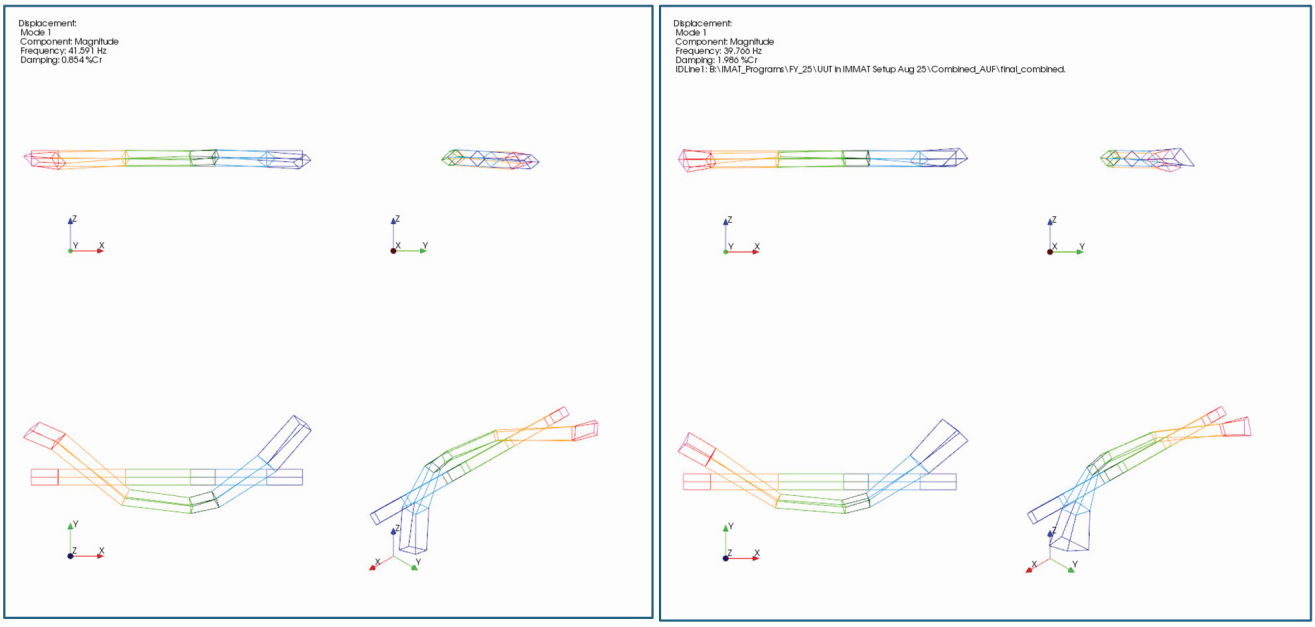


Fig. 3 Left: Free-Free: Mode 2. Right: Configured for IMMAT: Mode 2

SDT theme III (excitation strategy)

While SDT themes I, II, and IV are primarily analysis based, the Theme III Excitation Strategy requires infrastructure and test hardware. For replication of the captive carriage vibration environments, it is important to select an excitation strategy such as the IMMAT technique that provides distributed excitation while avoiding over-constraint of the test article's structural modes. With the inclusion of the IMMAT method in the pending release of MIL-STD-810J Method 527, it is important to understand the practical implementation and limitations associated with the IMMAT approach.

The RTC IMMAT infrastructure consists of eight electro-dynamic exciters and a reconfigurable framework from which the exciters could be mounted were integrated into an existing test facility. The random force ratings of the original suite of shakers range from approximately 120 to 500 lbf rms. The shaker displacements vary from ± 0.25 inch to ± 0.5 inch. Figure 1 illustrates the reconfigurable IMMAT platform in the five-shaker arrangement used for this example.

Shaker Setup

An IMMAT test configuration appears to look very similar to a multiple exciter modal test, however, there is a significant difference in that the shakers will be required to impart significantly higher acceleration levels to the UUT. For this reason, the stinger attachment between the shaker and UUT will need to be more robust and still allow relative motion in DOFs other than the drive direction without damaging the shaker itself and minimizing the effect on the overall mechanical impedance. This of course, is a premise of Theme II. The stingers employed for this example initially were a preloaded cable/strut design with an elastomer joint at each end that allows rotational and cross axis translational motion [7]. While these would have been sufficient for vibration up to 500 Hz, the losses between 500-2000 Hz required the struts to be replaced with solid threaded rods (5/8" diameter) to provide more efficient transfer of high frequency vibration into the UUT structure. Initial indications are that the threaded rods provide improved axial stiffness and sufficiently low rotational stiffness to protect the shaker armatures from side loading during test. Variations of the threaded rods, including reduction of the rod diameter, will continue to be assessed in an ongoing effort to maximize axial stiffness while minimizing rotational stiffness of connection.

Drive Points and Control Accelerometer Locations

A traditional right hand rule Cartesian coordinate system was used to define the motion degrees-of-freedom. The origin was considered to be the forward centerline tip of the UUT with +X assigned to the direction of flight and +Z upward. Triaxial accelerometers were placed along the length of the top of the UUT at the positions defined in Table 1.

Table 1 Accelerometer Locations

CH	Location	DOF	Type
1	1	+X	Control
2	1	+Y	Control
3	1	+Z	Control
4	2	+X	Control
5	2	+Y	Control
6	2	+Z	Control
7	3	+X	Control
8	3	+Y	Control
9	3	+Z	Control
10	4	+X	Control
11	4	+Y	Control
12	4	+Z	Control
13	5	+X	Measure
14	5	+Y	Measure
15	5	+Z	Measure
16	6	+X	Control
17	6	+Y	Control
18	6	+Z	Control
19	7	+X	Control
20	7	+Y	Control
21	7	+Z	Control

The accelerometer positions correlate directly to the positions from which field data was measured to develop the test reference profiles. For the initial trials, the drive points, as defined in Table 2, were selected to match the captive carry attachment points used in previous dual-shaker tests. Optimization of the drive points, a product of Theme IV, is planned for subsequent trials and will be discussed later in this document.

Table 2 Drive Point Locations

Drive	Location	DOF	Shaker Mdl
3	6	+X	V − 100
5	6	-Y	V − 300
6	4	-Y	V − 300
7	4	-Z	V − 350
8	6	-Z	V − 350

Strain Gage/Bending Moment Instrumentation

The use of tactically placed strain gage measurements during the field data acquisition phase of a test program is always highly recommended. The direct relationship between strain and fatigue and the ability to estimate moments imparted on the test item of interest can serve as key parameters in evaluating laboratory test fidelity. For the example UUT, strain gages were installed at six locations (three vertical and three lateral) and bending moment calibration was conducted by applying static loads to the structure at predetermined distance from the measurement points.

Formulation of the Spectral Density Matrix

As discussed for Theme I, ASD reference criteria in each of the three mutually orthogonal directions were provided at six locations along the length of the test item. Unfortunately, and is often the case, CSD profiles were not provided. Ideally, the product of Theme I, will be a fully populated SDM based on field measurements that would serve as the test reference criteria.

For the cases in which CSD terms are not provided, the approach outlined below is employed. It is essentially a minimum drive approach and is based on the assumption that the impedance match between field and laboratory is close and that the forcing function was uncorrelated between DOF's. The process below takes advantage of the Hermitian structure of the $n\ x\ n\ x\ d$ structure of the SDM, preserving the phase and scaling the magnitude as coherence squared.

Step I: Send a set of uncorrelated drive signals with a flat spectrum shape over the bandwidth of interest to each of the shakers and measure the drive and control point time histories.

Step II: Compute the following: Φ_{xx} (Cumulative Input Auto-Spectrum Estimate)
Φ_{yx} (Cumulative Cross-Spectrum Estimate)
H_{yx} (Transfer Function Estimate)

Step III: Compute $\Phi_{yy} = H_{yx}\Phi_{xx}H_{yx}^{*'}$

Step IV: Assign the spectral density matrix (SDM) as Φ_{yy} at each spectral line.

Step V: Compute normalized spectral density matrix (SDMn) by dividing each cross terms SDM(j,k) term at each spectral line d by $\sqrt{G_{jj}G_{kk}}$.

Step VI: Replace the diagonal (ASD) terms of SDMn with the Reference ASD terms for the test of interest.

Step VII: Convert SDMn back to SDM by multiplying each of the cross terms SDM(j,k) term by $\sqrt{G_{jj}G_{kk}}$ where the diagonal terms are now represented by the Auto-Spectrum of the desired Reference. This will yield the reference criteria for the test as a fully populated SDM.

Note that the cumulative (average) spectral estimates above will have errors associated with the number of averages taken, therefore estimates with 100 measurement DOF or more are recommended. Using the reference ASD's that were provided for each of the control points defined in Table 2 and the CSD development process outlined above, the natural coupling between DOFs will be preserved. Reference SDM's were established for the Buffet (10-500 Hz) and non-Buffet (10-2000 Hz) environments of interest. A frequency resolution of 1.25 Hz was selected for both cases. Multi-Shaker random testing was then conducted for each case. Time histories were captured during each trial run and processed posttest.

Trial Results

The results of the IMMAT trials are discussed in terms of three primary test objectives: 1) Modal Similarity, 2) Control of the Spectral Density Matrix, and 3) Bending Moment Characterization.

1) Modal Similarity

As a check to quantify the effects of the natural modal characteristics of the UUT while configured for the subject IMMAT test, a modal survey was conducted on the UUT in a free-free mode and again with the collar fixtures installed and attached to the freely suspended shakers as configured for the IMMAT test. Figures 2 & 3 illustrate the first bending modes of the highly symmetric UUT as computed for the free-free and the IMMAT configuration.

The Modal Assurance Criterion (MAC) matrix in Table 3 provides a comparison of the free-free modes to the modes of the UUT as configured for the IMMAT test. Observe that there is an approximate 5% reduction of the frequency of the first bending modes and an increase in estimated damping, both associated with the combination of the additional collar mass and attachment to the shakers. Very similar differences were also observed for the second and third bending modes.

2) Control of the Spectral Density Matrix

The initial 5-shaker IMMAT test configuration yielded strong similarities to the Buffet reference criteria across the 18 control channels of interest. Illustrating the SDM results for the 18-channel control scheme is difficult in the limited size of a traditional manuscript, so a reduced version illustrating only the dominant Z-Axis control channels is provided in Figure 4. The graphic takes advantage of the Hermitian nature of the SDM by plotting the ASD's down the diagonal with phase information on the lower triangular region and the associated coherence in the upper triangular region. The test yielded very similar results for the X and Y-Axis control channels as well.

Table 3 CMAC between free-free and IMMAT configured UUT

Cross-MAC Table											90 - 100
			Shapes (IMMAT)							Test	CRSS	50 - 89
			1	2	3	4	5	6	7			
MACtg	Shapes (free-free)		39.00	39.77	67.12	108.09	112.42	177.62	185.05	3%	All	0 - 49
100	1	41.02	96							99	99	
95	2	41.59		97						99	99	
90	3	68.69			93		38			96	98	
85	4	109.15				97				99	99	
80	5	114.89			43		97			98	99	
75	6	187.41						93		97	98	
70	7	197.40							95	97	98	
65	FEM	3%	99	99	96	99	98	97	97			
60	CRSS	All	99	99	98	99	99	98	98			

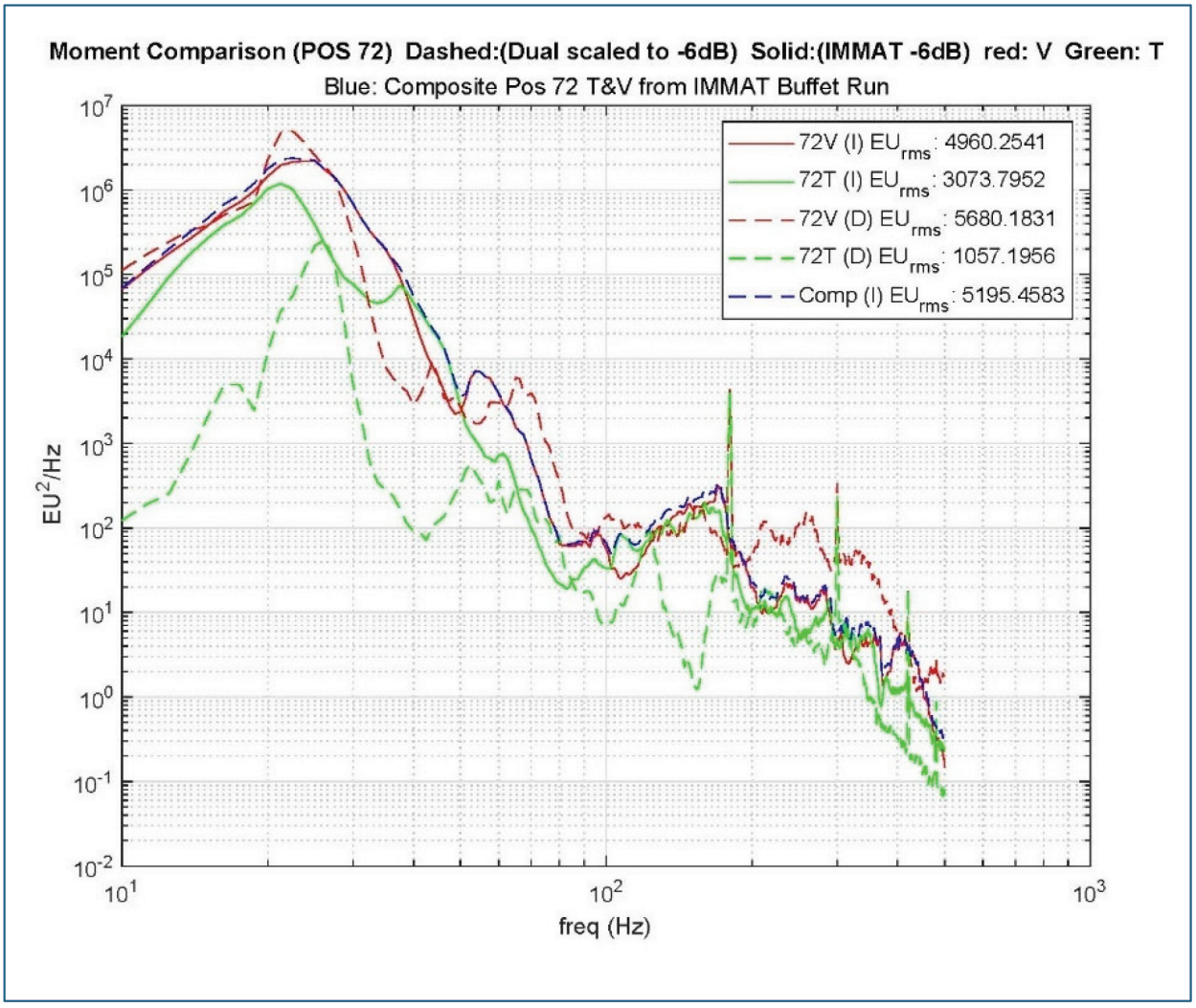


Fig. 4 SDM of the Z-Axis control channels. Red: Reference vs. Blue: IMMAT Trial

A compact review of the test results is shown in Figure 5 as a summation of the first five singular value ASD plots and the associated percentage of the total system response as provided in Table 4. The singular value ASDs are generated from taking a singular value decomposition of the full SDMs. This process is detailed in reference [8]. This compact view contains information from all the translational ASD's and associated CSD's. The associated singular vectors will contain estimates of the associated operational deflection shapes.

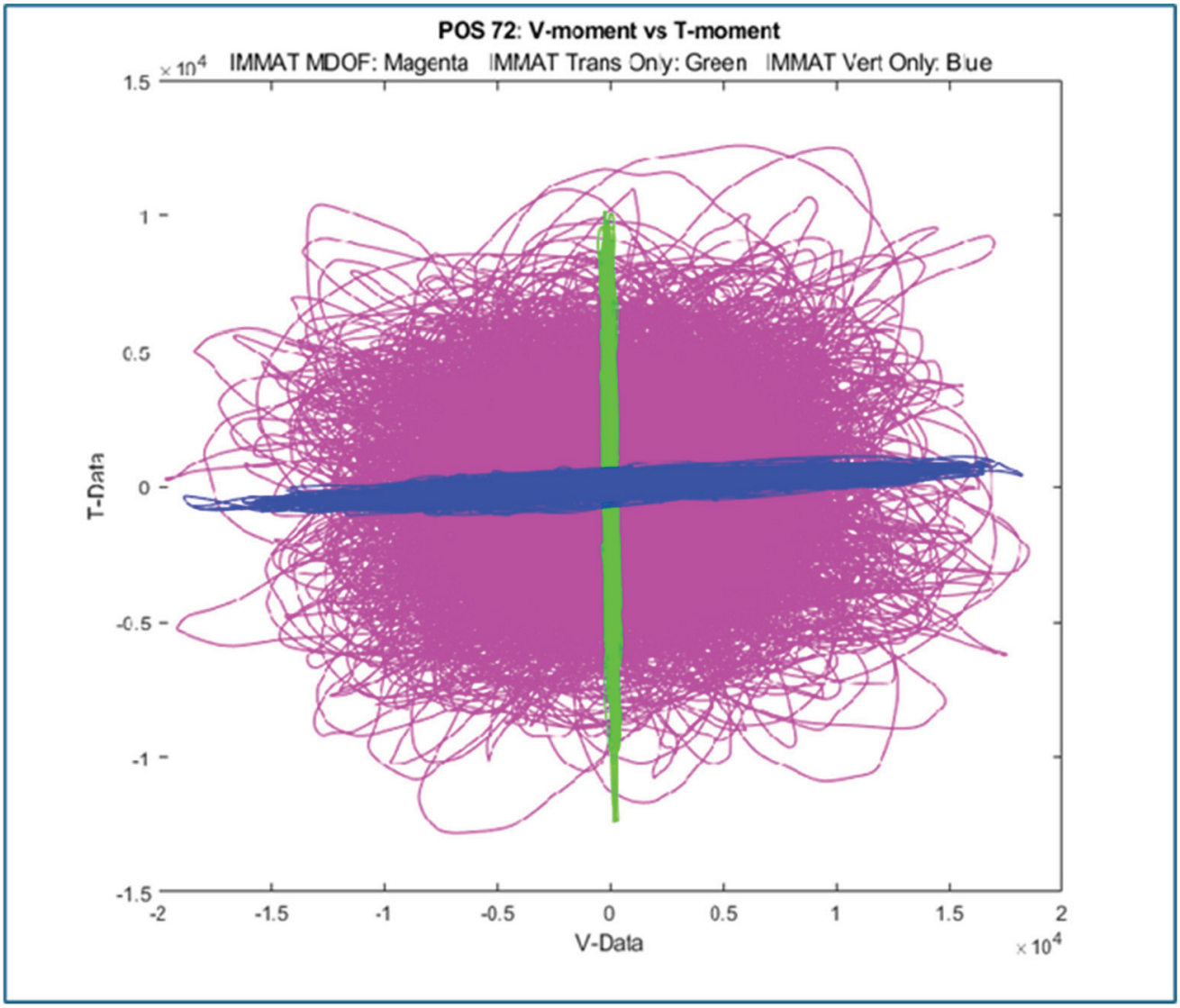


Fig. 5 Line by line sum of the five Singular Value ASD's (Left: Buffet Reference & Right: Non-Buffet)

Table 4 Tabular view of Laboratory test results relative to Reference Singular Value ASD RMS values

	Buffet		
	Lab2Ref (rms) %	**Ref2RefSum (rms) %**	**RunSum2RefSum (%)**
SV01	96.7	40.60	40.60
SV02	101	27.70	68.30
SV03	102	16.70	85.00
SV05	104	11.10	96.20
SV05	118	3.82	99.92

	Non-Buffet		
	Lab2Ref (rms) %	**Ref2RefSum (rms) %**	**RunSum2RefSum (%)**
SV01	71.60	40.60	40.60
SV02	67.70	25.40	66.00
SV03	66.30	16.10	82.10
SV05	63.50	11.40	93.50
SV05	67.90	6.50	100.00

The left-hand graphic in Figure 5 is associated with the Buffet vibration as was Figure 4. The right-hand graphic in Figure 5 is associated with the non-Buffet environment. Observe the wider test bandwidth and significantly different spectral shape of the non-Buffet reference. In the initial non-Buffet trials, it proved difficult to couple the higher frequency energy (> 1000 Hz) into the UUT with the initial shaker configuration as illustrated in the right-hand graph of Figure 5 and associated Table 4 results. The less-than-favorable results are also in part due to the specification variations between closely spaced control locations. This issue is typically an unintended artifact of the specification development process and may indicate a local response on the measurement vehicle that is not present in the test article (e.g., loose fins or internal components). In test, this requires some level of averaging or notching between the control locations which effectively reduces the overall similarity to the reference ASDs at these locations. The combination of difficulty in coupling the high frequency energy into the structure and the nature of the proposed reference criteria restricted the maximum attainable non-Buffet test level of the current test configuration to approximately -3 dB of the current reference criteria.

As vibration specifications are refined and models are developed for the example UUT, optimization studies will be conducted targeted at addressing the current deficiencies in addressing the higher frequency reference criteria of the non-Buffet environment.

3) Bending Moment Characterization

The bending moment test results presented below are based on the Buffet reference criteria since that is the scenario for which maximum bending moments are expected. For the subject UUT, as demonstrated above via modal analysis the key bending modes have been minimally affected by the test setup. Figure 6 illustrates measured moment comparisons between previous dual exciter vertical axis testing and the IMMAT trials. Observe that the shape of the moments induced by vertical axis motion and associated rms levels are comparable between previous dual shaker vertical trials in which the drive points correlated to the bomb rack attachment points, and the current 5-shaker IMMAT trials. Also observe that the IMMAT trials simultaneously yielded the moments induced by the transverse axis as well. The green dashed curve illustrates the cross-axis

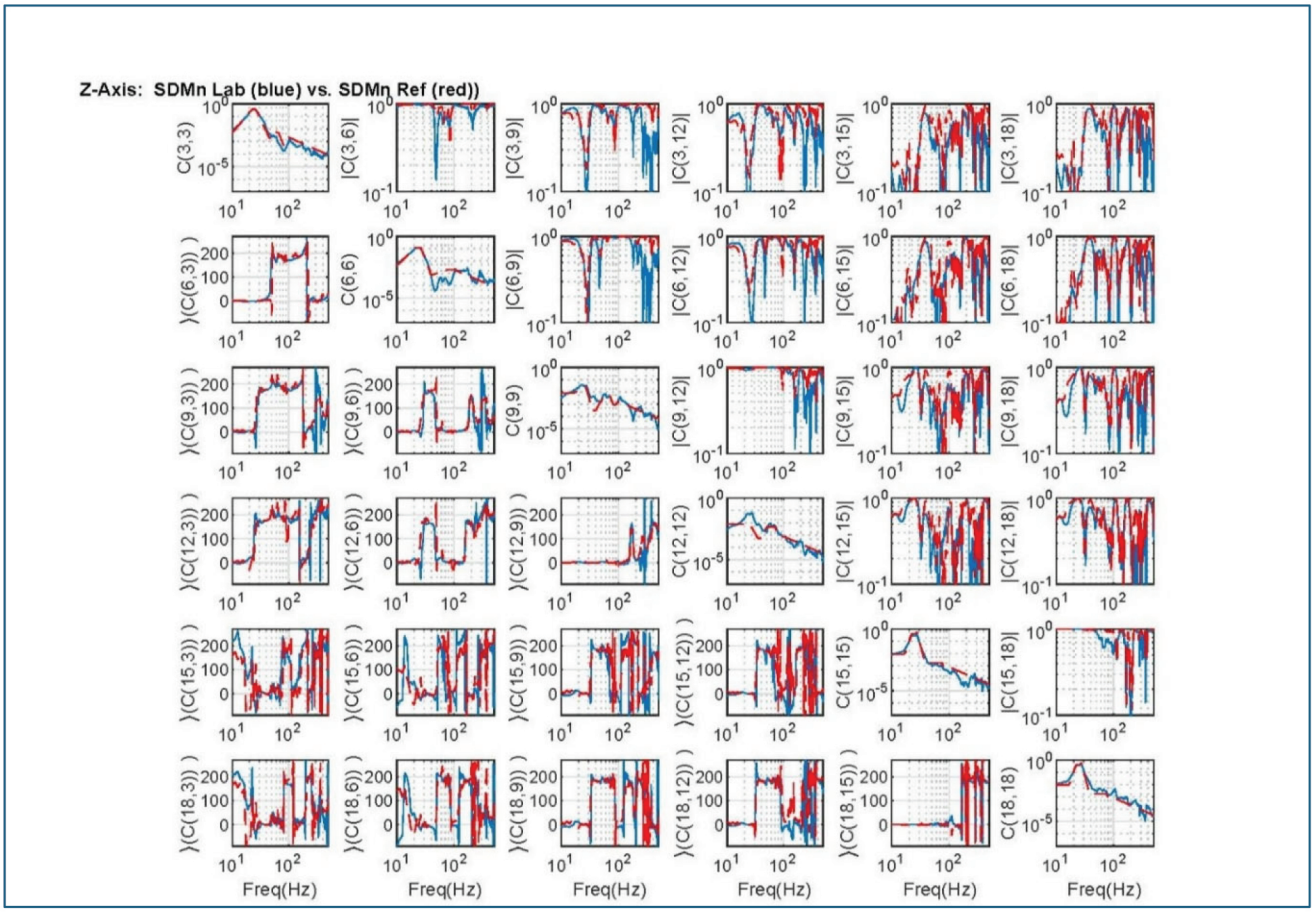


Fig. 6 Comparison of measured Moments between IMMAT and previous Z-axis only testing

moment measured during the vertical axis dual shaker trials. Note that the much smaller set of shakers employed in the IMMAT yield the expected bending moments.

Prior trials with the same UUT used in this example have been conducted with dual shakers to perform sequential axis vibration testing. Such trials have shown evidence of uneven joint wear associated with dwelling in a uniaxial state. Figure 7 illustrates through a scatter plot of bending moment time histories as measured during the 5-shaker IMMAT trial vs. exciting the structured in the Y and Z axes in a 1-DOF manner. While the peak bending moments in each axis are similar, observe the distribution of bending moment vectors that is clearly evident in the IMMAT approach, eliminating the aforementioned uneven wear issues associated with the 1-DOF approach.

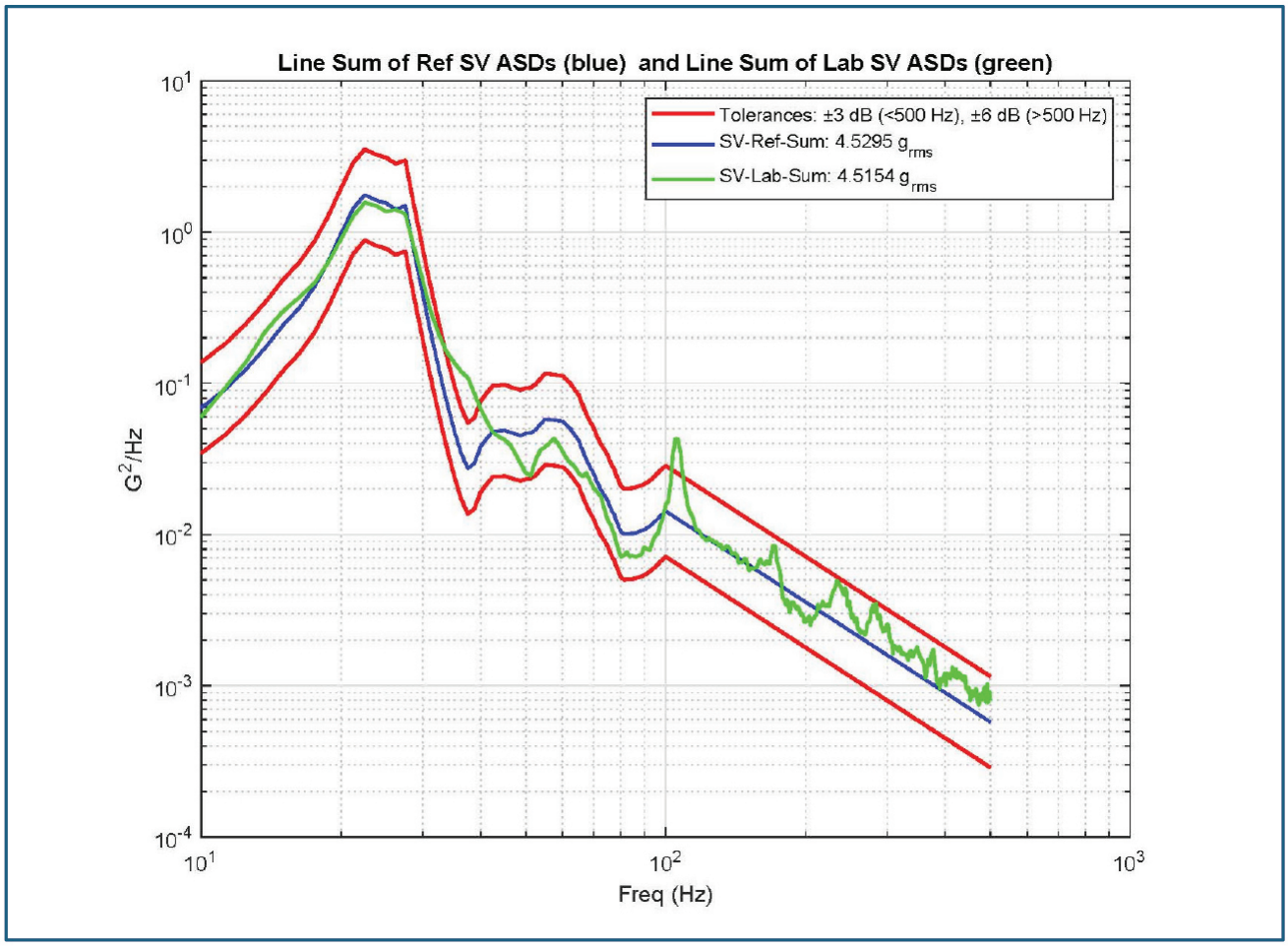


Fig. 7 Scatter of Moment Data Between IMMAT Approach and serial 1-DOF

Theme IV (Test Optimization and Integration)

The objectives of Theme IV are largely associated with identifying opportunities to improve the fidelity of the test by addressing gaps in the environment definition, test boundary conditions, and excitation strategy. Comprehensive analytical models that utilize modal characteristics of the system to identify optimal drive and control points have the potential to be valuable optimization tools but often require high fidelity models and a significant level of expertise. In the absence of validated finite element or other analytical models, iterative changes to the test setup and excitation strategy can help to improve correlation to the objective ASDs, but may also lead to costly mechanical changes that rely on engineering judgement as opposed to analytical assessments. As with many test scenarios, the tradeoff between analytical and iterative approaches is often dictated by cost and schedule constraints.

In the example presented, analytical models were not available to fully optimize the drive points into the UUT structure. Instead, initial attachments points were defined at locations corresponding to the launcher interfaces and the shaker sizes were initially based on engineering judgement. Several iterations in the stinger design were evaluated and some improvements were observed especially in the higher frequency transmissibility. Overall, satisfactory results were achieved with a reasonable number of iterations in the setup. However, deficiencies were identified that may require more substantial and costly changes (e.g., procurement of additional shakers) in order to achieve the defined higher level qualification environment(s). To address these deficiencies, analytical methods will be employed to mitigate the risk of ineffective solutions.

Conclusions

Clearly the implementation of the four themes of SDT takes considerable forethought. As demonstrated in the example employed in this document, it is still possible to conduct a quality IMMAT test without having all the desired information to fully address each of the four themes of SDT. Iterative approaches to test optimization are typical in the classical laboratory environment and expected to be so for IMMAT trials as well.

While initial trials have been limited to half-level vibration, the captive carriage example above yielded very favorable preliminary results for both the 10-500 Hz Buffet and the 10-2000 Hz non-Buffet vibration environments. Current projections are that both environments can be achieved up to approximately -3 dB, but some adjustments of the shaker configuration will be required to achieve full level. For both the buffet and non-buffet environments, spectral control in each axis at the six control points were in family with results from prior 1-DOF trials, but the ability to excite all axes simultaneously provides much more realistic loading of the UUT structure.

There are clear benefits that have been identified in the application of IMMAT testing techniques. As illustrated above in the Figure 7 discussion, issues such as uneven wear of joints associated with dwelling in a uniaxial state are avoided. While a properly planned IMMAT test allows the UUT to flex in a natural manner, it also yields the advantage of only requiring the test to be configured once and requires only one-third of the run-time as compared to traditional serial 1-DOF testing. This is very advantageous when dealing with a UUT that is being tested at temperature extremes with significant amount of instrumentation. IMMAT tests address the four themes of SDT well and allow rapid changes in setup to improve results.

Future Work

While the initial IMMAT trials discussed proved successful, there are still significant improvements possible that should yield additional fidelity. Specific planned improvements associated with the four SDT themes are as follows:

Theme I: Continue to highlight the need to develop IMMAT references that are highly correlated to the tactical deployment environment. Techniques such as significant sky lining of multiple events with dissimilar spectral characteristics should be avoided. To address such cases, multiple reference criteria are preferred and easy to implement once the test is configured.

Theme II: Design and integration of a lighter/reconfigurable collar assembly to provide greater flexibility in placement of shakers while minimizing added mass to the test setup. Design and integration of a stinger assembly with improved high frequency transmissibility characteristics is in progress.

Theme III: Integration of additional midsize shaker systems to address current test level limitations.

Theme IV: Build/acquire a FEM of the UUT and experiment with the use of ATA IMAT modal pre-test utilities to further optimize instrumentation and drive point selections [9].

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