Appendix A

Part A: Units and Conversions

International Units

mass	kg	kilogram	1 tonne = 1,000 kg
length	m	meter	
time	s	second	
force	N	Newton	N = kg m/s^2
energy	J	Joule	J = N m
power	W	Watt	W = J/s
temperature	°C or K	Celsius or Kelvin	Kelvin is absolute temperature
pressure	Pa	Pascal	Pa = N/m^2

International Unit Conversions and Constants

1 GJ = 277.78 kWh	1 Wh = 3.6 kJ	k = 10^3
1 kWh = 3,600 kJ = 3.6 MJ	1 kcal = 4.184 kJ	M = 10^6
1 kW = 3,600 kJ/hr = 3.6 MJ/hr	1,000 L = 1 m^3	G = 10^9

English Units

mass	lbm	pounds mass	
length	ft	foot	
time	s	second	
force	lbf	pounds force	$F_{gravity}$ = m g/g_c
energy	Btu	British thermal units	
power	Btu/hr	Btu per hour	
temperature	°F or R	Fahrenheit or Rankine	Rankine is absolute temperature
pressure	lbf/ft^2	pounds per square foot	1 psi = 1 lbf/in^2 = 144 lbf/ft^2 = 27.68 inch H_2O

English Unit Conversions and Constants

1 therm = 100,000 Btu	1 dekatherm (dth) = 1 MMBtu
1 ton = 2,000 lbm	1 ft^3 = 1,728 in^3
1 ton of thermal energy = 12,000 Btu/hr	g_c = 32.2 ft lbm / lbf s^2
1 ft^3 = 7.4805 US gallon	M = 1,000 = 10^3
1 ft^2 = 144 in^2	MM = 1,000,000 = 10^6
1 hp = 33,000 ft lbf / min	MBH = 1,000 Btu/hr
1 ft^3/s = 448.83 gpm [U.S. gallons per minute]	
Boiler Horsepower = 34.5 lbm/hr saturated steam @ 14.696 psia = 34.5 × 970.3 = 33,475 Btu/hr [output]	

Useful English-to-International Unit Conversions

1 hp = 0.7457 kW
1 lbm = 0.4536 kg
1 lbf = 4.448 N
1 m = 3.2808 ft
1 m^2 = 10.76 ft^2
1 m^3 = 35.315 ft^3
1 kWh = 3,600 kJ = 3,412 Btu
1.055 GJ = 1 MMBtu = 1 × 10^6 Btu
°C = (°F – 32) / 1.8
1 W/m^2 °C = 5.676 Btu/hr ft^2 °F or RSI = R / 5.676

Part B: Thermodynamics

Temperature: $\mathrm{K} = {}^{\circ}\mathrm{C} + 273.15$ [International units: Kelvin is absolute temperature scale]

Temperature: $\mathrm{R} = {}^{\circ}\mathrm{F} + 459.67$ [English units: Rankine is absolute temperature scale]

Pressure: $\mathrm{P_{gage}} = \mathrm{P_{abs}} - \mathrm{P_{atm}}$
International: kPag is gage pressure; kPa is absolute pressure
P_{atm} = 101.325 kPa = 101.325 kN/m^2 at sea level
P_{atm} = 0 kPa gage pressure at any elevation
1 bar = 100 kPa is *approximately* equal to one atmosphere of pressure at sea level

English: psig is gage pressure; psia is absolute pressure
Sea level: P_{atm} = 14.696 lbf/in^2 = 2,116 lbf/ft^2 = 406.78 inch H_2O = 14.696 psia

P_{atm} = 0.00 psig at any elevation

Water and Air at Standard Conditions
International: sea level pressure, temperature $\sim$ 20 °C
Water specific heat, C_p = 4.184 kJ / kg °C
Water density, ρ = 1.00 kg/L = 1,000 kg/m^3
Air density, ρ = 1.204 kg/m^3= 0.001204 kg/L
Air specific heat, C_p = 1.006 kJ / kg °C

English: sea level pressure, temperature $\sim$ 68 °F
Water specific heat, C_p = 1.00 Btu / lbm °F
Water density, ρ = 8.34 lbm / gal = 62.4 lbm/ft^3
Air density, ρ = 0.0752 lbm/ft^3
Air specific heat, C_p = 0.2403 Btu/ lbm °F

Quality of a pure substance (temperature-enthalpy or temperature specific volume charts)
x= quality = mass fraction of vapor for a pure substance, two phase mixture of liquid and vapor

$$\mathrm{h} = \mathrm{h_f} + x\mathrm{h_{fg}}$$

Enthalpy definition: $\mathrm{h} = \mathrm{u} + \mathrm{P}v$. From thermodynamics, u is the internal energy of a substance.

Conservation of Mass (steady state)
$\dot{m} = \rho\dot{\mathcal{V}} = \rho VA = \frac{VA}{v}$ and $\dot{\mathcal{V}} = VA$
steady state: $\dot{m}_{in} = \dot{m}_{out} = \dot{m}$
A = cross section area, ft^2.
$A_{square\ or\ rectangular}$ = (height)(width)
$A_{circular}$ = $(\pi/4)(diameter)^2$

International :
$\dot{m}$ = mass flow rate, kg/s
$\mathcal{V}$= volume, m^3
$\dot{\mathcal{V}}$= volume flow rates, m^3/s
v= specific volume, m^3/kg
ρ= density = $1/v$, kg/m^3
V = velocity, m/s

English :
$\dot{m}$ = mass flow rate, lbm/s
$\mathcal{V}$= volume, ft^3
$\dot{\mathcal{V}}$= volume flow rates, ft^3/s

v= specific volume, ft^3/lbm
ρ= density = $1/v$, lbm/ft^3
V = velocity, ft/s

Ideal gas law for air
p = ρRT

International:
p = pressure, kPa
R = gas constant = 0.2870 kJ / kg K = 0.2870 kPa m^3 / kg K for air
T = absolute temperature, K
ρ = density, kg/m^3

Example for air at 1 atm & 20°C:
ρ=p/RT = 101.325 kPa/[(0.2870 kPa m^3/kg K)(273.15+20)K] = 1.204 kg/ m^3

English:
p = pressure, lbf/ft^2
R = gas constant = 53.35 ft-lbf / lbm-R for air
T = absolute temperature, R
ρ = density, lbm/ft^3

Example for air at 1 atm & 68°F:
ρ=p/RT =2,116 lbf/ft^2 / [(53.35 ft-lbf/lbm-R)(459.67+68)R] = 0.0752 lbm/ ft^3

First Law of Thermodynamics (steady state conservation of energy)

- Energy is neither created nor destroyed. It only changes into different forms.
- In thermodynamics, thermal energy or heat is defined as energy that moves due to a temperature difference. Work is defined as all other forms of energy, other than heat.
- Suggested text: Cengel & Boles, "Thermodynamics: An Engineering Approach" (10$^{\text{th}}$ ed, 2023) McGraw Hill

First law of thermodynamics for steady state, steady flow control volume (open system):

$$\dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}_{in} \left(h_{in} + \frac{V_{in}^2}{2} + g z_{in} \right)$$

$$= \dot{Q}_{out} + \dot{W}_{out} + \sum_{out} \dot{m}_{out} \left(h_{out} + \frac{V_{out}^2}{2} + g z_{out} \right)$$

Steady state heating or cooling of air or water flow with no work, negligible change in elevation, and negligible change in velocity is a common situation. The first law simplifies to:

$$\dot{Q}_{in} - \dot{Q}_{out} = \dot{Q} = \dot{m}\ (h_{out} - h_{in}) = \dot{m}\ \Delta h$$

If there is only sensible heating or cooling (no latent heat involved: no moisture added, no moisture removed):

$$\Delta h = c_P\ \Delta T \text{ and } \dot{Q}_{in} - \dot{Q}_{out} = \mathbf{\dot{Q} = \dot{m}\ c_P\ \Delta T}$$

Steady state change in thermal energy (sensible + latent) of air and water vapour mixtures (psychrometrics)

$$\dot{Q} = \dot{m}\ \Delta h = \dot{\mathcal{V}}\ \rho\ 60\ \Delta h$$

International:

$\dot{Q}$ = thermal energy loss or gain, kJ/hr
$\dot{\mathcal{V}}$ = actual dry air flow, m^3 / min
ρ = density of dry air, kg / m^3 [psychrometric chart provides specific volume, v = 1 / ρ]
60 = minutes per hour
Δh = change in enthalpy, kJ / kg of dry air [from psychrometric chart]

English:

$\dot{Q}$ = thermal energy loss or gain, Btu/hr
$\dot{\mathcal{V}}$ = actual dry air flow, ft^3 / min
ρ = density of dry air, lbm / ft^3 [psychrometric chart provides specific volume, v = 1 / ρ]
60 = minutes per hour
Δh = change in enthalpy, Btu/lbm of dry air [from psychrometric chart]

Air sensible only (no latent) thermal energy change

$$\dot{Q} = \dot{\mathcal{V}}\ \rho\ C_p\ \Delta T$$

International :

$\dot{Q}$ = sensible energy change, kW
$\dot{\mathcal{V}}$ = volume rate of air flow, m^3/s [note: 1 m^3/s = 1000 L/s]
ρ = 1.204 kg/m^3 = density of air at standard conditions
C_p = 1.006 kJ / kg °C = specific heat of air at standard conditions
ΔT = temperature difference, °C

$$\dot{Q} = 1.21\ \dot{\mathcal{V}}\ \Delta T$$

English :

$\dot{Q}$ = sensible energy change, Btu/hr
$\dot{\mathcal{V}}$ = volume rate of air flow, ft^3/hr [note: ft^3/hr = ft^3/min × 60 min/hr]
ρ = 0.0752 lbm/ft^3 = density of air at standard conditions
C_p = 0.2403 Btu / lbm °F = specific heat of air at standard conditions
ΔT = temperature difference, °F
$\dot{Q} = 0.0180\ \dot{\mathcal{V}}\ \Delta T$
If flow in ft^3/min: $\dot{Q} = 1.08\ \dot{\mathcal{V}}\ \Delta T$

Water sensible only (no latent) thermal energy change

$$\dot{Q} = \dot{\mathcal{V}}\ \rho\ C_p\ \Delta T$$

International:

$\dot{Q}$ = sensible energy change, kW
$\dot{\mathcal{V}}$ = volume rate of water flow, L/s [note: 1 m^3/s = 1000 L/s]
ρ = 1.00 kg/L = density of water at standard conditions
C_p = 4.2 kJ / kg °C = specific heat of water at standard conditions
ΔT = temperature difference, °C
$\dot{Q} = 4.2\ \dot{\mathcal{V}}\ \Delta T$

English:

$\dot{Q}$ = sensible energy change, Btu/hr
$\dot{\mathcal{V}}$ = volume rate of water flow, ft^3/hr
note: ft^3/hr = ft^3/min × 60 min/hr and U.S. gal/min = ft^3/min × 7.48 gal/ft^3
ρ = 62.4 lbm/ft^3 = density of water at standard conditions
C_p = 1.00 Btu / lbm °F = specific heat of water at standard conditions
ΔT = temperature difference, °F
$\dot{Q} = 62.4\ \dot{\mathcal{V}}\ \Delta T$
If flow in U.S. gal/min: $\dot{Q} = 500.5\ \dot{\mathcal{V}}\ \Delta T$

Heating and Cooling Degree Days

International:

- DD units are: °C day / unit time [unit of time is often 1 yr, but not always]
- Reference temperature is 18.3 °C for ***heating*** or ***cooling*** unless otherwise specified
- Example: if average outdoor is 10 °C for 1 week then HDD = (18.3 - 10) × 7 = 58.1 °C day / week

English:

- DD units are: °F day / unit time [unit of time is often 1 yr, but not always]
- Reference temperature is 65 °F for ***heating*** and ***cooling*** unless otherwise specified
- Example: if average outdoor is 45 °F for 1 week then HDD = (65 - 45) × 7 = 140 °F day / week

Annual energy loss (heating) or gain (cooling) due to thermal energy transfer through envelope

$$\dot{Q} = U \;\; A \;\; DD \;\; 24$$

International:

$\dot{Q}$ = useful energy loss or gain, Wh / yr [note this is Wh, not kWh]
U = overall thermal conductance, W / m^2 °C
A = heat transfer area, m^2
DD = annual heating or cooling degree-days, °C day / yr
24 = hr/day

English:

$\dot{Q}$ = useful energy loss or gain, Btu/yr
U = overall thermal conductance, Btu / ft^2 hr °F
A = heat transfer area, ft^2
DD = annual heating or cooling degree-days, °F day / yr
24 = hr/day

Energy loss or gain due to air mass loss or gain through a hole or crack (*infiltration or exfiltration*)

$$\dot{Q} = \dot{\mathcal{V}} \;\; 1440 \;\; \rho \;\; C_p \;\; (HDD + CDD)$$

International:

$\dot{Q}$ = useful energy loss or gain, kJ/yr
$\dot{\mathcal{V}}$ = actual m^3 / min of dry air flow
1440 = number of minutes per day
ρ = density of air at standard conditions = 1.204 kg/ m^3
C_p = specific heat of air at standard conditions = 1.006 kJ / kg °C
HDD = annual heating degree-days, °C day / yr
CDD = annual cooling degree-days, °C day / yr

English:

$\dot{Q}$ = useful energy loss or gain, Btu/yr
$\dot{\mathcal{V}}$ = actual ft^3 / min of dry air flow
1440 = number of minutes per day
ρ = density of air at standard conditions = 0.0752 lbm/ ft^3
C_p = specific heat of air at standard conditions = 0.2403 Btu / lbm °F
HDD = annual heating degree-days, °F day / yr
CDD = annual cooling degree-days, °F day / yr

Efficiency of Fuel Combustion:
Natural gas: $\eta_{HHV} = 0.91 \times \eta_{LHV}$
No. 2 Fuel Oil (diesel fuel): $\eta_{HHV} = 0.94 \times \eta_{LHV}$

Performance of vapour compression and other HVAC equipment
International:
COP = useful thermal energy delivered / input energy [dimensionless]
air conditioner or heat pump in cooling mode: $COP_{max} = T_{cold} / (T_{hot} - T_{cold})$
heat pump in heating mode: $COP_{max} = T_{hot} / (T_{hot} - T_{cold})$
note: must use absolute temperatures (Kelvin) in the COP_{max} formulas

$$\frac{kW_{input}}{kW_{thermal}} = \frac{kWh_{input}}{kWh_{thermal}} = \frac{1}{COP}$$

English:
EER = energy efficiency ratio
= useful thermal energy in Btu delivered per Wh input energy [Btu per Watt-hour]

COP is unitless. See International unit formulas above.

$$\frac{kW_{input}}{ton_{thermal}} = \frac{kWh_{input}}{ton.h_{thermal}} = \frac{12}{EER} = \frac{3.517}{COP}$$

HSPF = heating season performance factor = season heat output divided by Wh of energy input
HSPF = Btu of useful energy delivered per Wh input energy [Btu / Wh]
Seasonal average COP = HSPF / 3.412 = HSPF × 0.2931

Per 2024 ASHRAE Handbook—HVAC Systems and Equipment (SI), HSPF and EER are only defined in English units.

"ton" of thermal energy = 12,000 Btu/hr = energy to melt one ton of ice (2,000 lbm @ 32°F) in 24 hr

- ice heat of fusion @ 32°F = 144 Btu/lbm (335 kJ/kg @ 0 °C)

- 2,000 lbm × 144 Btu/lbm = 288,000 Btu
- 288,000 Btu / 24 hr = 12,000 Btu/hr

Part C: Fluid Mechanics

- Liquids and gases are described as "fluids"
- Basic 'law' of fluid mechanics: **fluids flow from high pressure to low pressure**

Energy Balance Equation for Fluids

- Fluid energy includes three terms: pressure, gravity, and kinetic energy
- Fluid energy is expressed in "head" which is energy per unit weight. English units are ft-lbf/lbf = ft. International units are N-m/N = m.
- Key assumptions: incompressible flow, steady state, neglect heat & other energy forms

Bernoulli's Energy Balance Equation (using words)
(initial fluid energy) – (pipe or duct friction losses) – (minor losses such as valves & bends)
+ (energy added by pumps) – (energy extracted by turbines) = (final fluid energy)

Bernoulli's Energy Balance Equation (using math)

$$\frac{p_1}{\gamma} + z_1 + \frac{V_1^2}{2g} - f\frac{L}{D}\frac{V^2}{2g} - K_{\mathrm{L}}\frac{V^2}{2g} + H_{\text{pump}} - H_{\text{turbine}} = \frac{p_2}{\gamma} + z_2 + \frac{V_2^2}{2g}$$

Subscript 1 = initial condition (upstream location along a streamline)
Subscript 2 = final condition (downstream location along a streamline)

International units:

p = pressure, kPa
z = height of fluid, m
g = gravity constant, 9.81 m/s^2
V = average velocity of fluid, m/s
f = dimensionless friction factor from Moody diagram (see any fluid mechanics text book)
L = length of pipe, m
D = internal diameter of pipe, m
K_L = dimensionless minor loss factor; lookup in table (see any fluid mechanics text book)

H_{pump} = head added by pump, m
$H_{turbine}$ = head extracted by turbine, m
γ = the specific weight of the fluid, N/m^3. Standard conditions:
γ_{water} = 9,810 N/m^3
γ = sg γ_{water}
sg = specific gravity for a liquid = γ of actual liquid /
γ_{water} = dimensionless

English units:
p = pressure, lbf/ft^2
z = height of fluid, ft
g = gravity constant, 32.2 ft/s^2
V = average velocity of fluid, ft/s
f = dimensionless friction factor from Moody diagram (see any fluid mechanics text book)
L = length of pipe, ft
D = internal diameter of pipe, ft
K_L = dimensionless minor loss factor; lookup in table (see any fluid mechanics text book)
H_{pump} = head added by pump, ft
$H_{turbine}$ = head extracted by turbine, ft
γ = the specific weight of the fluid, lbf/ft^3. Standard conditions:
γ_{water} = 62.4 lbf/ft^3
γ = sg γ_{water}
sg = specific gravity for a liquid =
γ of actual liquid / γ_{water} = dimensionless

Fluid power added by pumps or extracted by turbines

International units:
pumps: $P_{fluid} = \gamma \dot{\mathcal{V}} H_{pump}$ / 1000 is fluid power *added* by pump
turbines: $P_{fluid} = \gamma \dot{\mathcal{V}} H_{turbine}$ / 1000 is fluid power *extracted* by a turbine
P_{fluid} = fluid power, kW
γ = specific weight, N/m^3 = sg $\times$ γ_{water} [γ_{water} = 9,810 N/m^3]
$\dot{\mathcal{V}}$ = volume flow rate of liquid, m^3/s
H_{pump} = fluid head added by pump, m
$H_{turbine}$ = fluid head extracted by turbine, m

English units:
pumps: $P_{fluid} = \gamma \dot{\mathcal{V}} H_{pump}$ / 550 is fluid power *added* by pump
turbines: $P_{fluid} = \gamma \dot{\mathcal{V}} H_{turbine}$ / 550 is fluid power *extracted* by a turbine

P_{fluid} = fluid power = hp
γ = lbf/ft^3 = sg $\times$ γ_{water} [γ_{water} = 62.4 lbf/ft^3]
$\dot{\mathcal{V}}$ = volume flow rate of liquid, ft^3/s
H_{pump} = fluid head added by pump, ft
$H_{turbine}$ = fluid head extracted by turbine, ft
Common form of the above formula when $\dot{\mathcal{V}}$ is in U.S. gal/min:

$$P_{fluid} = sg\ \dot{\mathcal{V}}\ H_p / 3{,}956$$

Fluid power added by fans

International units:
P_{fluid} = ΔTP $\dot{\mathcal{V}}$ / 60
P_{fluid} = air or gas fluid power, kW
ΔTP = fan total pressure rise, kPa
ΔTP is total fluid pressure added by the fan, approximated by fan static pressure from a fan curve
$\dot{\mathcal{V}}$ = actual air or gas flow, m^3/min
60 = seconds per minute

English units:
P_{fluid} = ΔTP $\dot{\mathcal{V}}$ / 6,343
P_{fluid} = air or gas fluid power, hp
ΔTP = fan total pressure rise, inches of water
ΔTP is total fluid pressure added by the fan, approximated by fan static pressure from a fan curve
$\dot{\mathcal{V}}$ = actual air or gas flow, ft^3/min
6,343 = unit conversion constant

Part D: Heat Transfer

- Heat or thermal energy is energy in nature that flows due to a temperature difference.
- Basic 'law' of heat transfer: **heat (thermal energy) flows from high temperature to low temperature.**

Convection mode:

$$\dot{Q}_{conv} = h_{conv}\ A\ (T_h - T_c) = \frac{A}{R_{conv}}\ (T_h - T_c)$$

International units:
$\dot{Q}_{conv}$ = convective heat flow, W

h_{conv} = convective or film coefficient, W / m^2 °C
[note: h is also used for enthalpy]
A = surface area of solid surface to fluid interface, m^2
T_h = hot temperature, °C or K
T_c = cold temperature, °C or K
$R_{conv} = 1/h_{conv}$ = convective film resistance, m^2 °C / W

English units:
$\dot{Q}_{conv}$ = convective heat flow, Btu/hr
h_{conv} = convective or film coefficient, Btu / hr ft^2 °F
[note: h is also used for enthalpy]
A = surface area of solid surface to fluid interface, ft^2
T_h = hot temperature, °F or R
T_c = cold temperature, °F or R
$R_{conv} = 1/h_{conv}$ = convective film resistance, hr ft^2 °F / Btu

Conduction mode:

$$\dot{Q}_{cond} = \frac{k\,A}{t}\,(T_h - T_c) = \frac{A}{R_{cond}}\,(T_h - T_c)$$

International units:
$\dot{Q}_{cond}$ = conductive heat flow, W
k = thermal conductivity, W / m °C
L = material thickness, m [note: 1000 mm = 100 cm = 1 m]
A = cross section area of heat flow, m^2
T_h = hot temperature, °C or K
T_c = cold temperature, °C or K
$R_{cond} = L / k$ = conductive resistance, m^2 °C / W
Metric conduction resistance is usually described as RSI.
$C = 1/R_{cond}$ is thermal conductance for solids, W / m^2 °C

English units:
$\dot{Q}_{cond}$ = conductive heat flow, Btu/hr
k = thermal conductivity, Btu / hr ft °F
L = material thickness, ft
A = cross section area of heat flow, ft^2
T_h = hot temperature, °F or R
T_c = cold temperature, °F or R
$R_{cond} = L / k$ = conductive resistance, hr ft^2 °F / Btu
$C = 1/R_{cond}$ is thermal conductance for solids, Btu / hr ft^2 °F

Radiation mode:

$$\dot{\mathrm{Q}}_{\mathrm{rad}} = \mathrm{A}\ \left(T_{\mathrm{h}}^4 - T_{\mathrm{c}}^4\right) = \frac{\mathrm{A}}{\mathrm{R}_{\mathrm{rad}}}\left(\mathrm{T_h} - \mathrm{T_c}\right)$$

International units:

$\dot{\mathrm{Q}}_{\mathrm{rad}}$ = radiative heat flow, W

ϵ = thermal emissivity, dimensionless $0 \leq \epsilon \leq 1$

σ = Stefan-Boltzmann constant = 5.67×10^{-8} W / m^2 K^4

A = cross section area of heat flow, m^2

$\mathrm{T_h}$ = hot temperature, K. **Must use absolute temperature for radiation**. Do not use °C

$\mathrm{T_c}$ = cold temperature, K. **Must use absolute temperature for radiation**. Do not use °C

Radiation resistance is seldom used due to the 4$^{\mathrm{th}}$ power mathematics. But if temperatures are known, Radiative resistance equation is:

$\mathrm{R}_{\mathrm{rad}} = \frac{1}{\left(\mathrm{T_h^2} + \mathrm{T_c^2}\right)\left(\mathrm{T_h} + \mathrm{T_c}\right)}$ m^2 K / W

English units:

$\dot{\mathrm{Q}}_{\mathrm{rad}}$ = radiative heat flow, Btu/hr

ϵ = thermal emissivity, dimensionless $0 \leq \epsilon \leq 1$

σ = Stefan-Boltzmann constant = 0.1713×10^{-8} Btu / hr ft^2 R^4

A = cross section area of heat flow, ft^2

$\mathrm{T_h}$ = hot temperature, R. **Must use absolute temperature for radiation**. Do not use °F

$\mathrm{T_c}$ = cold temperature, R. **Must use absolute temperature for radiation**. Do not use °F

Radiation resistance is seldom used due to the 4$^{\mathrm{th}}$ power mathematics. But if temperatures are known, Radiative resistance equation is:[1]

$\mathrm{R}_{\mathrm{rad}} = \frac{1}{\left(\mathrm{T_h^2} + \mathrm{T_c^2}\right)\left(\mathrm{T_h} + \mathrm{T_c}\right)}$ hr ft^2 R / Btu

Multi-mode thermal resistor model for heat transfer:

$$\dot{\mathrm{Q}}_{\mathrm{total}} = \frac{\mathrm{A}}{\sum R}\left(\mathrm{T_h} - \mathrm{T_c}\right) = \mathrm{U\ A}\left(\mathrm{T_h} - \mathrm{T_c}\right)$$

International units:

$\sum R$ = sum of all heat transfer resistance values, m^2 °C / W

[1] $\mathrm{R}_{\mathrm{rad}}$ is derived using the math relation: $\left(\mathrm{T_h^4} - \mathrm{T_c^4}\right) = \left(\left(\mathrm{T_h^2} + \mathrm{T_c^2}\right)\left(\mathrm{T_h} + \mathrm{T_c}\right)\right)\left(\mathrm{T_h} - \mathrm{T_c}\right)$

$U = \frac{1}{\sum R}$ is defined as *overall thermal conductance*, W / m^2 °C

English units:

$\sum R$ = sum of all heat transfer resistance values, hr ft^2 °F / Btu

$U = \frac{1}{\sum R}$ is defined as *overall thermal conductance*, Btu / hr ft^2 °F

Heat exchangers:

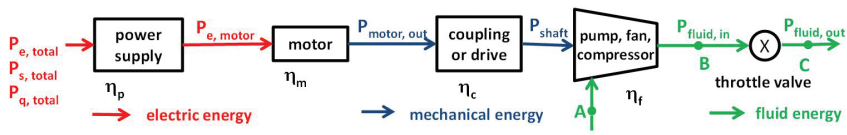
Heat exchanger effectiveness = $\dot{Q}_{actual}$ / $\dot{Q}_{max}$

$\dot{Q}_{actual}$ = actual heat transfer

$\dot{Q}_{max}$ = maximum heat transfer

Part E: Electric power and equipment

This part provides the common, approximate formulas for estimating power quantities in fluid moving systems (pumps, fans, compressors) powered by electric motors (applicable to Y or delta alternating current designs). The simplified diagram illustrates the typical arrangement.



Variables (International units only):

P_e = real or active power kW. Real kWh used for billing purposes by the utility

P_s = total or apparent power, kVA. Peak kVA often used for demand charges by the utility

P_q = reactive power, kVAr

PF = power factor [usually lagging power factor; PF $<$ 1 for inductive loads; PF = 1 for resistive loads]

$P_{e,\ motor}$ = real electric power input to the motor, kW

$P_{motor,\ out}$ = mechanical power out of the motor, kW note: **motors are always rated as <u>output</u> power**

$T_{motor,\ out}$ = torque delivered by the motor, Nm

$P_{motor,\ nameplate}$ = nominal, full load[2] output power, kW. Nameplate is usually stated in hp; convert to kW.

[2] "Full load" here is defined as motor loaded at its nameplate value. This does not necessarily equal full load for pumps, fans, or compressors.

P_{shaft} = power input to pump, fan, or compressor, kW
P_{fluid} = power added to increase pressure and move liquids and gases[3], kW.
$I_{nameplate}$ = full load motor active or "absorbed" current, from nameplate, amps
I_L = measured line current input to the system, amps
V_L = measured line voltage input to the system, Volts
ω = motor or fluid mover shaft rotational speed, rev/min or RPM
MLF = motor load factor = operating load / rated load
η_p = electric power supply efficiency [e.g. transformer or variable frequency drive]
η_m = motor efficiency [varies substantially with operating conditions such as motor load]
η_c = coupling or drive efficiency [e.g. belt drive]
η_f = fluid mover efficiency[4] [compressor, pump or fan; efficiency varies substantially based on design and operating conditions]

Electric and Mechanical Power Equations

1. $P_s = (3)^{1/2} \; I_L \; V_L$ / 1000 for 3-phase AC systems
2. $P_s = I_L \; V_L$ / 1000 for 1-phase AC systems
3. $P_e = I_L \; V_L$ / 1000 for DC systems
4. $P_s^2 = P_e^2 + P_q^2$
5. $PF = P_e / P_s$
6. $P_{motor,\,out} = (\omega \; T_{motor,\,out})$ / 9,550
   In English units: $P_{motor,\,out} = (\omega \; T_{motor,\,out})$ / 5,245 in units of hp, with torque in units of ft-lbf
7. $P_{e,\,total} = P_{e,motor} / \eta_p$
8. $P_{e,motor} = P_{motor,\,out} / \eta_m$
9. $P_{motor,\,out} = P_{shaft} / \eta_c$
10. $P_{shaft} = P_{fluid} / \eta_f$
11. $MLF = P_{motor,\,out} / P_{motor,\,nameplate}$
12. $P_{motor,\,out} = T_{motor,\,out} \; \omega_{meas}$ / 9550

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[3] Pfluid can also be calculated using fluid head term, Hpump or Hfan, from the Bernoulli equation: Pfluid = (specific weight) x (flow rate) x (fluid head). Pump manufacturers usually exclude valve losses from pump curves, and include valve losses in system curves. Fan manufacturers usually include inlet vanes or control damper losses in fan curves, and exclude vane or control damper losses from system curves.

[4] This is the conservation of energy law from physics, ratio of output to input energy. It is not the same as isentropic efficiency or other ways of quantifying efficiency for pumps, fans, or compressors.

13. $P_{shaft2} = P_{shaft1}\ (\omega_2/\omega_1)^n$
    n = 3 is affinity "law" for centrifugal systems
    is only true when static head or gravity forces are negligible (i.e. kinetic energy dominates)
14. $P_{e2,\ total} = P_{e1,\ total}\ (\omega_2/\omega_1)^n$
    n ≈ 1.8 to 2.8 when static head or gravity dominate for centrifugal systems
15. MLF can be determined using three different measurements (current, shaft speed, or electric power)
    ≈ $(I_{L,motor}\ /I_{nameplate})$ $(V_{L,motor}\ /V_{nameplate})$ is less accurate and should only use **when it is known that MLF > 50%**
    ≈ $(\omega_{sync}$ - $\omega_{meas})/(\omega_{sync} - \omega_{nameplate})$
    ≈ $(P_{motor,\ in})/(P_{motor,in\ @full\ load})$

Part F: Solar Energy

Solar PV

NOCT = normal operating cell temperature
P_{mp} = maximum power point (point on I-V curve where load is drawing maximum power), Watts
P_{DC} = direct current power, Watts (W_{DC})
P_{AC} = alternating current power, Watts (W_{AC})
I_{SC} = short circuit current, amps
I_{mp} = current which corresponds to P_{mp}, amps
I_{DC} = direct current, amps
I_{AC} = alternating current, amps
V_{OC} = open circuit voltage, Volts
V_{DC} = direct current voltage, Volts
V_{AC} = alternating current voltage, Volts
$T_{ambient}$ = environment or air temperature surrounding the PV module
T_{cell} = PV cell temperature
S = solar radiation, kWh/m^2

$$T_{cell} = T_{ambient} + \frac{NOCT - 20}{0.80} S$$

Solar Thermal

$$\dot{Q}_{useful\ heat} = \dot{Q}_{absorbed\ solar} - \dot{Q}_{lost\ to\ ambient}$$
$$= \alpha\tau AI - AU\,(T_{collector} - T_{ambient})$$

α = absorptivity coefficient
τ = transmissivity coefficient
A = area, m^2
I = solar irradiation, W/m^2
U = heat loss coefficient, $W/m^2\ ^\circ C$
$T_{collector}$ = temperature of the collector, °C
$T_{ambient}$ = temperature of the ambient, °C
$\dot{Q}_{useful\ heat}$ = rate of useful heat available from the collector, W
$\dot{Q}_{absorbed\ solar}$ = rate of heat absorption by the collector, W
$\dot{Q}_{lost\ to\ ambient}$ = rate of heat loss to the ambient, W

$$\dot{Q}_{useful\ heat} = \dot{m} c_p \left(T_{fluid\ out} - T_{fluid\ in}\right)$$

$\dot{m}$ = mass flow rate, kg/s
c_p = fluid heat capacity, kJ/kg°C
$T_{fluid\ out}$ = temperature output fluid
$T_{fluid\ in}$ = temperature input fluid

$$\eta_{collector} = \frac{\dot{Q}_{useful\ heat}}{AI} = \alpha\tau - U\frac{T_{collector} - T_{ambient}}{I}$$

$\eta_{collector}$ = collector's efficiency

$$\eta_{collector} = F_R\alpha\tau - F_R U\frac{T_{fluid\ in} - T_{ambient}}{I}$$

F_R = heat removal factors